2.2 Angles Formed by Parallel Lines

GOAL

Prove properties of angles formed by parallel lines and a transversal, and use these properties to solve problems.

YOU WILL NEED

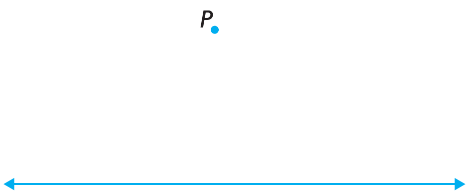
- compass
- protractor
- ruler

INVESTIGATE the Math

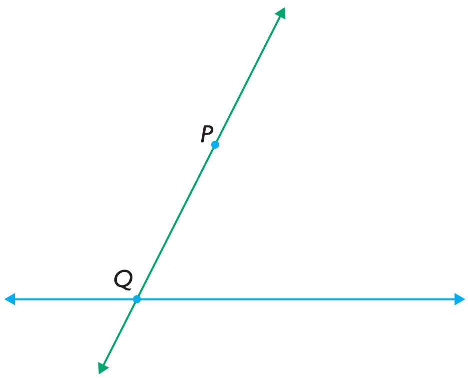
Briony likes to use parallel lines in her art. To ensure that she draws the parallel lines accurately, she uses a straight edge and a compass.

? How can Briony use a straight edge and a compass to ensure that the lines she draws really are parallel?

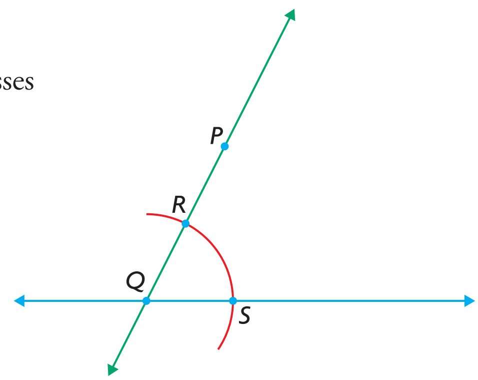
A. Draw the first line. Place a point, labelled *P*, above the line. *P* will be a point in a parallel line.



B. Draw a line through *P*, intersecting the first line at *Q*.



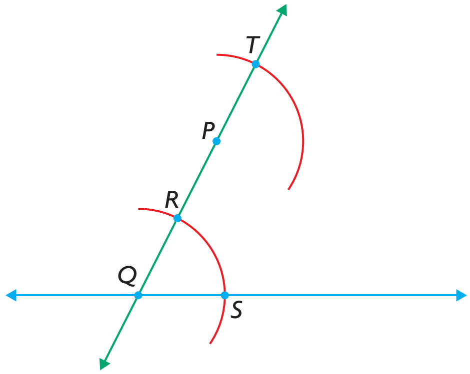
C. Using a compass, construct an arc that is centred at *Q* and passes through both lines. Label the intersection points *R* and *S*.



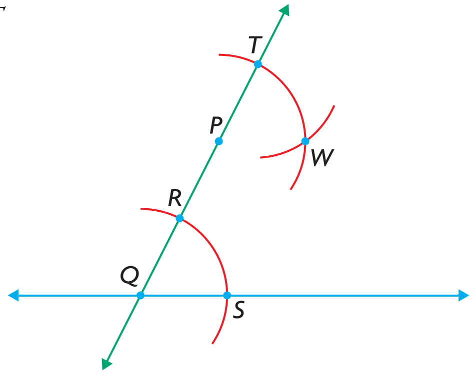
EXPLORE...

- Parallel bars are used in therapy to help people recover from injuries to their legs or spine. How could the manufacturer ensure that the bars are actually parallel?

D. Draw another arc, centred at P, with the same radius as arc RS. Label the intersection point T.



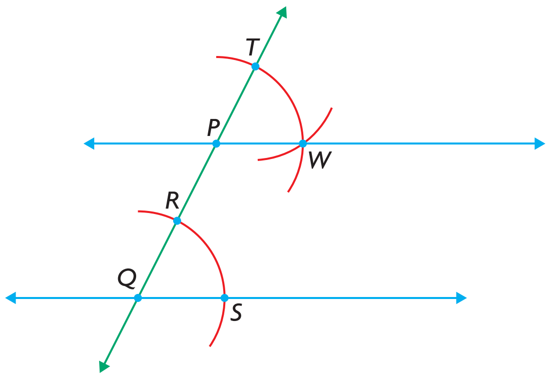
E. Draw a third arc, with centre T and radius RS, that intersects the arc you drew in step D. Label the point of intersection W.



Communication | *Tip*

If lines *PW and QS* are parallel, you can represent the relationship using the symbol ‖ :
PW ‖ *QS*

F. Draw the line that passes through P and W. Show that $PW \parallel QS$.



Reflecting

G. How is $\angle SQR$ related to $\angle WPT$?

H. Explain why the compass technique you used ensures that the two lines you drew are parallel.

I. Are there any other pairs of equal angles in your construction? Explain.

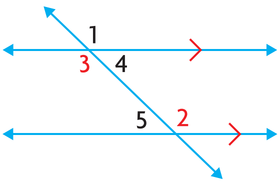
APPLY the Math

EXAMPLE 1 Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Tuyet's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the **alternate interior angles** are equal.



I drew two parallel lines and a transversal as shown, and I numbered the angles. I need to show that $\angle 3 = \angle 2$.

Statement	Justification
$\angle 1 = \angle 2$	Corresponding angles
$\angle 1 = \angle 3$	Vertically opposite angles
$\angle 3 = \angle 2$	Transitive property

Since I know that the lines are parallel, the corresponding angles are equal.

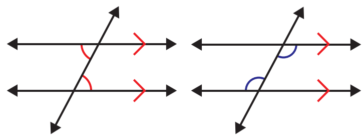
When two lines intersect, the opposite angles are equal.

$\angle 2$ and $\angle 3$ are both equal to $\angle 1$, so $\angle 2$ and $\angle 3$ are equal to each other.

My conjecture is proved.

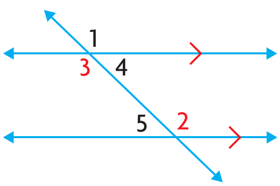
alternate interior angles

Two non-adjacent interior angles on opposite sides of a transversal.



Ali's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the interior angles on the same side of the transversal are supplementary.



I need to show that $\angle 3$ and $\angle 5$ are supplementary.

$$\angle 1 = \angle 2$$

Since the lines are parallel, the corresponding angles are equal.

$$\angle 2 + \angle 5 = 180°$$

These angles form a straight line, so they are supplementary.

$\angle 1 + \angle 5 = 180°$

Since $\angle 2 = \angle 1$, I could substitute $\angle 1$ for $\angle 2$ in the equation.

$\angle 1 = \angle 3$

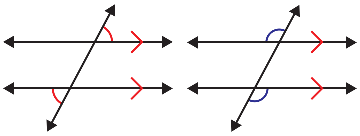
Vertically opposite angles are equal. Since $\angle 1 = \angle 3$, I could substitute $\angle 3$ for $\angle 1$ in the equation.

$\angle 3 + \angle 5 = 180°$

My conjecture is proved.

alternate exterior angles

Two exterior angles formed between two lines and a transversal, on opposite sides of the transversal.

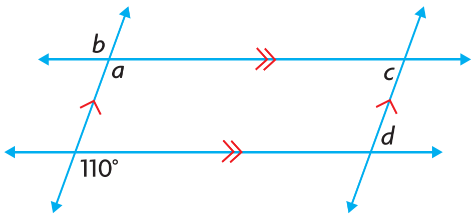


Your Turn

Naveen made the following conjecture: "**Alternate exterior angles** are equal." Prove Naveen's conjecture.

EXAMPLE 2 Using reasoning to determine unknown angles

Determine the measures of *a*, *b*, *c*, and *d*.



Kebeh's Solution

$\angle a = 110°$

The 110° angle and $\angle a$ are corresponding. Since the lines are parallel, the 110° angle and $\angle a$ are equal.

$\angle a = \angle b$

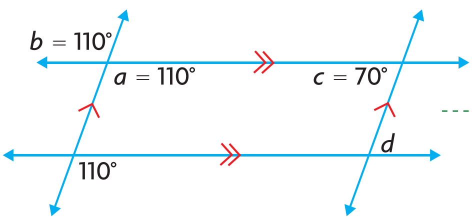
$\angle b = 110°$

Vertically opposite angles are equal.

$\angle c + \angle a = 180°$

$\angle c + 110° = 180°$

$\angle c = 70°$



$\angle c$ and $\angle a$ are interior angles on the same side of a transversal. Since the lines are parallel, $\angle c$ and $\angle a$ are supplementary.

I updated the diagram.

$\angle c = \angle d$

$\angle d = 70°$

$\angle c$ and $\angle d$ are alternate interior angles. Since the lines are parallel, $\angle c$ and $\angle d$ are equal.

The measures of the angles are:

$\angle a = 110°$; $\angle b = 110°$;

$\angle c = 70°$; $\angle d = 70°$.

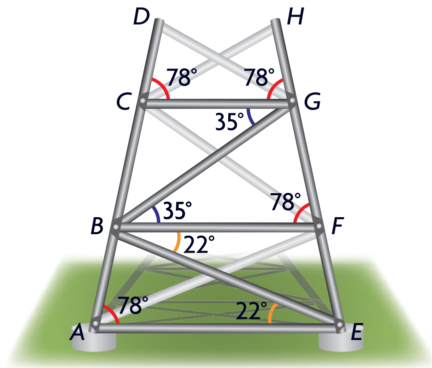
Your Turn

a) Describe a different strategy you could use to determine the measure of $\angle b$.

b) Describe a different strategy you could use to determine the measure of $\angle d$.

EXAMPLE 3 Using angle properties to prove that lines are parallel

One side of a cellphone tower will be built as shown. Use the angle measures to prove that braces *CG*, *BF*, and *AE* are parallel.



Morteza's Solution: Using corresponding angles

$\angle BAE = 78°$ and $\angle DCG = 78°$ — Given

$AE \parallel CG$ — When corresponding angles are equal, the lines are parallel.

$\angle CGH = 78°$ and $\angle BFG = 78°$ — Given

$CG \parallel BF$ — When corresponding angles are equal, the lines are parallel.

$AE \parallel CG$ and $CG \parallel BF$ — Since *AE* and *BF* are both parallel to *CG*, all three lines are parallel to each other.

The three braces are parallel.

Jennifer's Solution: Using alternate interior angles

Statement	Justification
$\angle CGB = 35°$ and $\angle GBF = 35°$	Given
$CG \parallel BF$	Alternate interior angles
$\angle FBE = 22°$ and $\angle BEA = 22°$	Given
$BF \parallel AE$	Alternate interior angles
$CG \parallel BF$ and $BF \parallel AE$	Transitive property

When alternate interior angles are equal, the lines are parallel.

When alternate interior angles are equal, the lines are parallel.

Since *CG* and *AE* are both parallel to *BF*, they must also be parallel to each other.

The three braces are parallel.

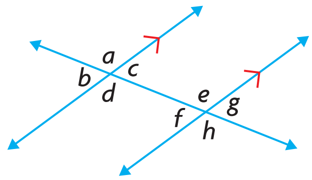
Your Turn

Use a different strategy to prove that *CG*, *BF*, and *AE* are parallel.

In Summary

Key Idea

- When a transversal intersects two parallel lines,
  - i) the corresponding angles are equal.
  - ii) the alternate interior angles are equal.
  - iii) the alternate exterior angles are equal.
  - iv) the interior angles on the same side of the transversal are supplementary.



i) $a = e, b = f$
$c = g, d = h$

ii) $c = f, d = e$

iii) $a = h, b = g$

iv) $c + e = 180°$
$d + f = 180°$

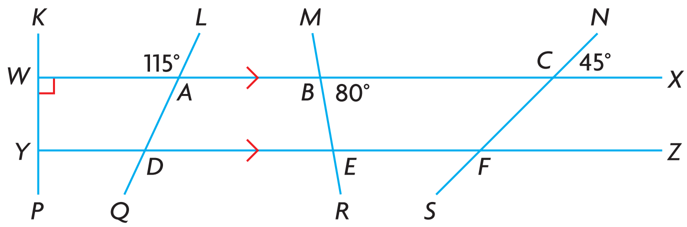
Need to Know

- If a transversal intersects two lines such that
  - i) the corresponding angles are equal, or
  - ii) the alternate interior angles are equal, or
  - iii) the alternate exterior angles are equal, or
  - iv) the interior angles on the same side of the transversal are supplementary,

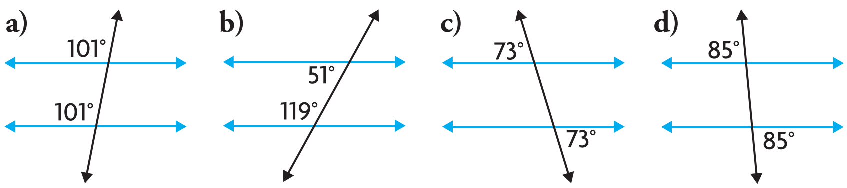
  then the lines are parallel.

CHECK Your Understanding

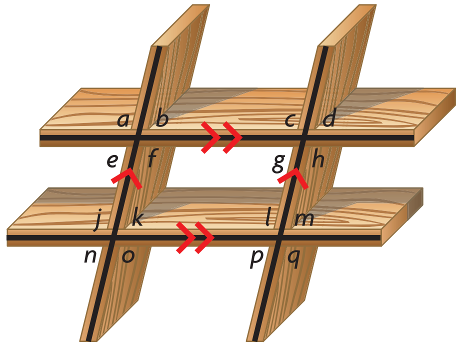
1. Determine the measures of $\angle WYD$, $\angle YDA$, $\angle DEB$, and $\angle EFS$. Give your reasoning for each measure.



2. For each diagram, decide if the given angle measures prove that the blue lines are parallel. Justify your decisions.



PRACTISING

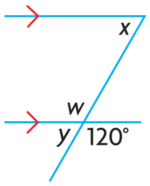


3. A shelving unit is built with two pairs of parallel planks. Explain why each of the following statements is true.

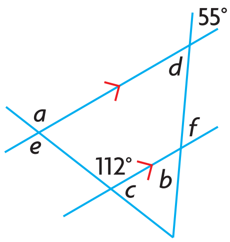
a) $\angle k = \angle p$
b) $\angle a = \angle j$
c) $\angle j = \angle q$
d) $\angle g = \angle d$
e) $\angle b = \angle m$
f) $\angle e = \angle p$
g) $\angle n = \angle d$
h) $\angle f + \angle k = 180°$

4. Determine the measures of the indicated angles.

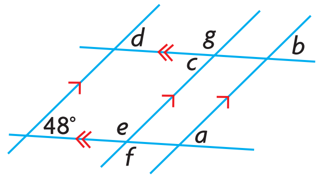
a)



b)



c)



5. Construct an isosceles trapezoid. Explain your method of construction.

6. a) Construct parallelogram *SHOE*, where $\angle S = 50°$.
b) Show that the opposite angles of parallelogram *SHOE* are equal.

7. a) Identify pairs of parallel lines and transversals in the embroidery pattern.
b) How could a pattern maker use the properties of the angles created by parallel lines and a transversal to draw an embroidery pattern accurately?

8. a) Joshua made the following conjecture: "If $AB \perp BC$ and $BC \perp CD$, then $AB \perp CD$." Identify the error in his reasoning.

Joshua's Proof

Statement	Justification
$AB \perp BC$	Given
$BC \perp CD$	Given
$AB \perp CD$	Transitive property

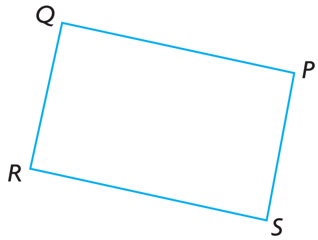
b) Make a correct conjecture about perpendicular lines.

Embroidery has a rich history in the Ukrainian culture. In one style of embroidery, called *nabiruvannia*, the stitches are made parallel to the horizontal threads of the fabric.

The Bank of China tower has a distinctive 3-D shape. The base of the lower part of the building is a quadrilateral. The base of the top is a triangle, making it stable and wind-resistant.

9. The Bank of China tower in Hong Kong was the tallest building in Asia at the time of its completion in 1990. Explain how someone in Hong Kong could use angle measures to determine if the diagonal trusses are parallel.

10. Jason wrote the following proof. Identify his errors, and correct his proof.



Given: $QP \perp QR$

$QR \perp RS$

$QR \parallel PS$

Prove: $QPSR$ is a parallelogram.

Jason's Proof

Statement	Justification
$\angle PQR = 90°$ and $\angle QRS = 90°$	Lines that are perpendicular meet at right angles.
$QP \parallel RS$	Since the interior angles on the same side of a transversal are equal, QP and RS are parallel.
$QR \parallel PS$	Given
$QPSR$ is a parallelogram	$QPSR$ has two pairs of parallel sides.

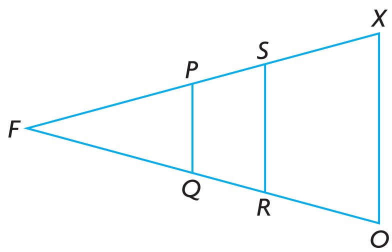
11. The roof of St. Ann's Academy in Victoria, British Columbia, has dormer windows as shown. Explain how knowledge of parallel lines and transversals helped the builders ensure that the frames for the windows are parallel.

12. Given: $\triangle FOX$ is isosceles.

$\angle FOX = \angle FRS$

$\angle FXO = \angle FPQ$

Prove: $PQ \parallel SR$ and $SR \parallel XO$



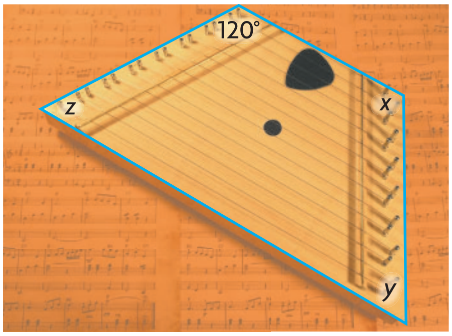
13. a) Draw a triangle. Construct a line segment that joins two sides of your triangle and is parallel to the third side.

b) Prove that the two triangles in your construction are similar.

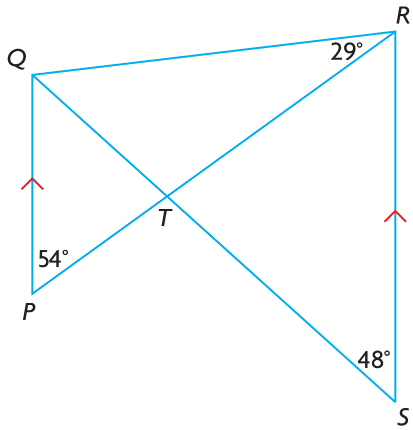
14. The top surface of this lap harp is an isosceles trapezoid.

a) Determine the measures of the unknown angles.

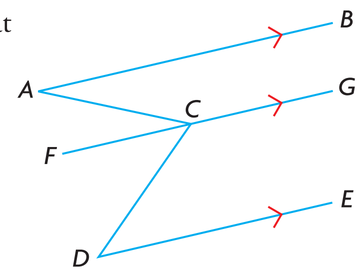
b) Make a conjecture about the angles in an isosceles trapezoid.



15. Determine the measures of all the unknown angles in this diagram, given $PQ \parallel RS$.



16. Given $AB \parallel DE$ and $DE \parallel FG$, show that $\angle ACD = \angle BAC + \angle CDE$.



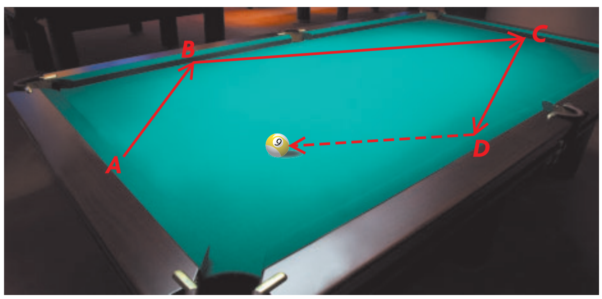
17. When a ball is shot into the side or end of a pool table, it will rebound off the side or end at the same angle that it hit (assuming that there is no spin on the ball).

a) Predict how the straight paths of the ball will compare with each other.

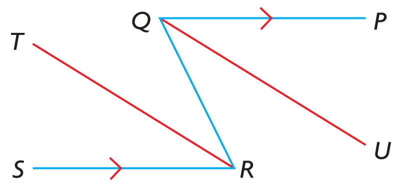
b) Draw a scale diagram of the top of a pool table that measures 4 ft by 8 ft. Construct the trajectory of a ball that is hit from point *A* on one end toward point *B* on a side, then *C*, *D*, and so on.

c) How does path *AB* compare with path *CD*? How does path *BC* compare with path *DE*? Was your prediction correct?

d) Will this pattern continue? Explain.



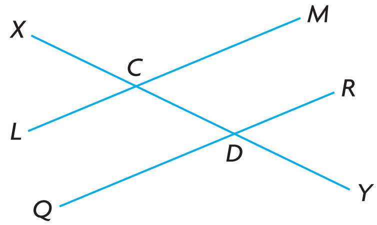
18. Given: $QP \parallel SR$
RT bisects $\angle QRS$.
QU bisects $\angle PQR$.
Prove: $QU \parallel RT$



Closing

19. a) Ashley wants to prove that $LM \parallel QR$. To do this, she claims that she must show all of the following statements to be true:
i) $\angle LCD = \angle CDR$
ii) $\angle XCM = \angle CDR$
iii) $\angle MCD + \angle CDR = 180°$
Do you agree or disagree? Explain.

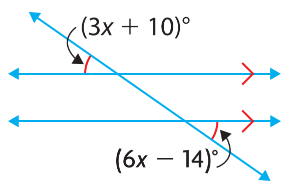
b) Can Ashley show that the lines are parallel in other ways? If so, list these ways.



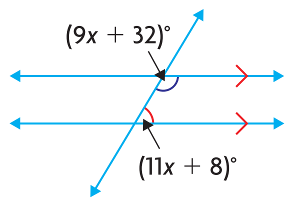
Extending

20. Solve for x.

a)



b)



21. The window surface of the large pyramid at Edmonton City Hall is composed of congruent rhombuses.

a) Describe how you could determine the angle at the peak of the pyramid using a single measurement without climbing the pyramid.

b) Prove that your strategy is valid.

The Edmonton City Hall pyramids are a city landmark.

Applying Problem-Solving Strategies

Checkerboard Quadrilaterals

One strategy for solving a puzzle is to use inductive reasoning. Solve similar but simpler puzzles first, then look for patterns in your solutions that may help you solve the original, more difficult puzzle.

The Puzzle

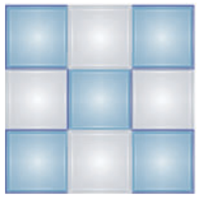
How many quadrilaterals can you count on an 8-by-8 checkerboard?

The Strategy

A. How many squares or rectangles can you count on a 1-by-1 checkerboard?

B. Draw a 2-by-2 checkerboard. Count the quadrilaterals.

C. Draw a 3-by-3 checkerboard, and count the quadrilaterals.



D. Develop a strategy you could use to determine the number of quadrilaterals on any checkerboard. Test your strategy on a 4-by-4 checkerboard.

E. Was your strategy effective? Modify your strategy if necessary.

F. Determine the number of quadrilaterals on an 8-by-8 checkerboard. Describe your strategy.

G. Compare your results and strategy with the results and strategies of your classmates. Did all the strategies result in the same solution? How many different strategies were used?

H. Which strategy do you like the best? Explain.